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Separation in oscillating laminar boundary-layer flows

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The results of an experimental investigation of separation in oscillating laminar boundary layers is reported. Instantaneous velocity profiles obtained with multiple hot-wire anemometer arrays reveal that the onset of wake formation is preceded by the initial vanishing of shear at the wall, or reverse flow, throughout the entire cycle of oscillation. Correlation of the experimental data indicates that the frequency, Reynolds number and dynamic history of the boundary layer are the dominant parameters and oscillation amplitude has a negligible effect on separation-point displacement.

Introduction

Various aspects of boundary-layer flows produced by streams having large-scale oscillations in velocity have been the subject of recent investigations, both analytic, as Lighthill (1954) and Nickerson (1957) and experimental, as Hill (1958) and Miller & Fejer (1964). Nevertheless, the important aspect of separation in such flows has not been adequately treated. The present study is an experimental investigation of separation in laminar boundary layers produced by a free stream having a large amplitude oscillating component of velocity.

Separation in oscillating flows

The importance of separation in steady flow lies in the fact that its occurrence, as characterized by the initial disappearance of shear at the wall, marks the beginning of a process that ultimately results in a streamline displacing region of wake flow. The essential feature of this process is the development of upstream fluid convection, or reverse flow, in the boundary layer, resulting in the production of eddies which characterize wake formation. To be relevant, any process or phenomenon defined as separation in oscillating flow, must be similarly related to the formation of a region of wake flow.

In oscillating flows, the steady flow description of separation is no longer meaningful. In such flows the character of the boundary layer at any spatial location varies as a function of time with the cyclic component of the flow. Hence, no single velocity profile can be associated with a given point on the surface. Moore (1958) predicts that instantaneous 'zero wall shear' profiles can occur when the flow downstream bears no evidence of displacement. Stuart (1963) calls attention to the fact that in oscillating flows, it is possible to observe both instantaneous 'zero wall shear' and 'reverse flow' profiles even in the absence of any mean pressure gradients.

Established techniques for the analysis of oscillating laminar boundary layers have in general proved to be inadequate for the treatment of flows in the presence of strong adverse pressure gradients. The analysis of the laminar boundary layer on a cylinder in oscillating flow of Lighthill (1954) fails to predict the flow soon after the pressure gradient on the body becomes positive. Similarly Hori's (1963) analysis of this problem diverges markedly from his experimental results in the region of adverse pressure gradient. Hill (1958) analytically treated the case of oscillations superimposed on a Howarth mean flow and experimentally confirmed his solution for one relatively small adverse pressure gradient. Nevertheless, he does not consider his analysis valid in the neighbourhood of separation.

Moore (1955, 1958) and more recently Hartunian & Moore (1959) have made some preliminary analysis of separation in unsteady laminar flows. These studies suggest that in the general case of a quasi-steady flow the problem may be divided into two associated problems, one which involves a non-steady perturbation of the quasi-steady flow and has a separation point fixed with respect to the wall and a second which involves a moving wall in steady motion. Unfortunately these efforts failed to attain their goal of determining a meaningful definition of separation in unsteady flow. Hartunian & Moore did, however, without rigorous proof, predict the general effects of local accelerations and decelerations on the boundary-layer profiles, and conjectured that separation might be characterized by a simultaneous vanishing of velocity and shear at some point in the boundary layer. Perhaps the most significant feature of this work is that it dramatically illustrates the difficulty of analytically arriving at a definition of separation in unsteady flow, as well as pointing out the requirement for such a definition. Rott (1964) also comments on the problem of defining separation in unsteady flow in a recent survey of laminar flow phenomena.

Vidal (1959) attempted to verify experimentally and to extend the work of Hartunian & Moore. Unfortunately this work was prematurely terminated, and although the efforts showed promise of some success, they must, on the whole be considered inconclusive.

Quite recently Sandborn (1969) has compiled a review of current concepts of boundary-layer separation. The phenomena of both laminar and turbulent separation under the influence of a variety of free-stream conditions is examined and attempts are made to develop an analytic model applicable to all forms of separation. Owing to the lack of experimental results, the case of laminar separation in an oscillating free stream is only qualitatively treated.

The primary objective of the present work is the determination of a rational definition for separation in oscillatory boundary-layer flows. A secondary objective is the correlation of the occurrence of separation in such flows with parametric variation enabling reasonable engineering predictions of separation to be made. In order to accomplish these objectives extensive measurements of instantaneous velocity profiles in oscillating laminar boundary layers subject to adverse pressure gradients have been made.

Experimental apparatus

The work described was carried out in a low-speed open-circuit wind tunnel having a 2 ft. square test section and a velocity range of 10–250 ft./sec. Mean free-stream turbulence intensities of 0·261 to 0·413 % were measured in this velocity range. Oscillations in the free-steam velocity were introduced by means of a rotating shutter valve (figure 1, plate 1) located between the test section and the diffuser.

The shutter-valve is similar to that described by Miller & Fejer (1964), and consists of a frame supporting four shafts equally spaced across the section height. Each shaft is provided with a slot into which flat blades of various widths may be introduced, forming a set of four butterfly valves spanning the cross-section. The range of blade widths employed allow perturbations from 6 to 80 % of the free-stream velocity to be introduced. The valve shafts are driven in synchronism through timing belts by a variable speed electric motor producing frequencies from 2 to 240 c/s.

The model employed in the study is an elongated two-dimensional symmetric air-foil with a 42 in. chord and thickness ratio of 0·0351. It was fabricated of mahogany laid up over aluminum spars and spans the entire test section. From the leading edge the first 6 in. of the profile consists of a half-ellipse terminating at the minor axis which is 1·5 in. long. The following 15 in. consists of a constant 1·5 in. thick section. The aft 21 in. is a straight taper to a trailing edge of 0·0625 in. which is faired into the constant thickness section. The upper surface has two 3 in. wide mild steel plates let into the surface which serve as traversing tracks for the hot-wire anemometer probe which is held in place by magnets. Seventeen static pressure taps are located in the upper surface, distributed from leading to trailing edge in a line 4 in. from the edge of the model. The model surface was given multiple coats of synthetic varnish and hand rubbed to a mirror finish to eliminate all surface discontinuities. The model is shown in place in figure 2 (plate 1).

Control of local pressure gradient was accomplished by altering the contour of the upper surface of the test section over the model. This was done by fastening shaped styrofoam blocks to the upper test section surface. Three different distributions of mean static pressure were investigated in an effort to assess the effects of boundary-layer dynamic history. These are shown in figure 3, in terms of the measured mean pressure coefficient. Each of the pressure distributions was designed to provide a significant chordwise region of favourable pressure gradient for the purpose of introducing separation inhibiting characteristics into the boundary layer. This was done in an effort to spatially extend, and thereby facilitate the observation of, the development of boundary-layer separation characteristics.

The Security Associates constant temperature hot-wire anemometers used for the velocity measurements consist of transistorized d.c.-coupled circuits having a measured frequency response of 50 kc. A self-contained transistorized analog computer serves to invert King's law and yields an output which is linear in velocity. The d.c. coupling permits the output to be separated into steady and oscillating components which is not otherwise possible. The d.c. components of the outputs were measured with the precision voltmeters integral to the units while the a.c. components were displayed on a Tektronix dual-beam oscilloscope and measured with a Ballantine true r.m.s. meter. In all, eleven channels of hot-wire anemometry were employed; as many as eight channels could be simultaneously displayed on the oscilloscope.

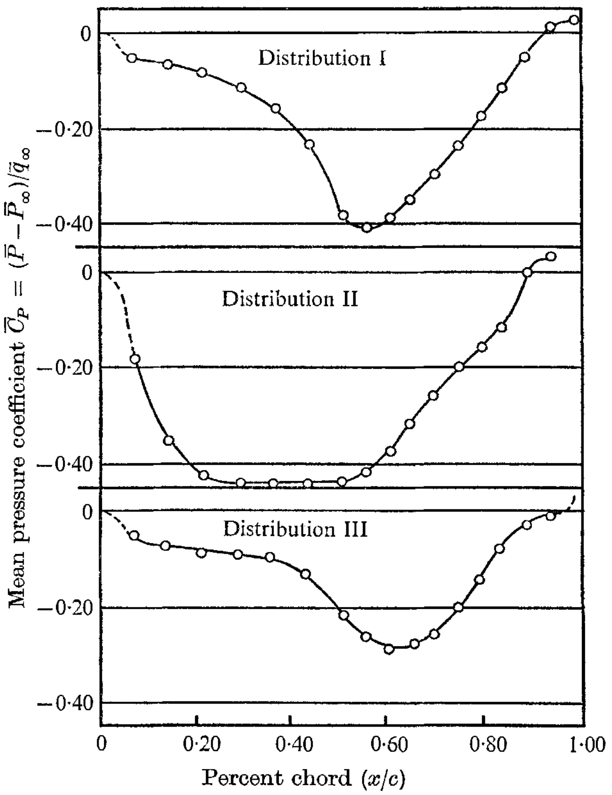

FIGURE 3. Mean pressure coefficient distributions.

Ten fixed hot-wire probes and one vertically traversable hot-wire probe were mounted on a chordwise traversable carriage. The ten probe fixed array consisted of five modular subarrays which could be arranged to cover depths from 0·150 to 0·400 in. normal to the model surface and which occupied 1·4 in. in the spanwise direction. The movable probe was remotely controlled by an external micrometer through a flexible shaft. Tungsten wire having a diameter of 0·00015 in. and an active length of 0·080 in. was employed in each of the probes. Calibration of the hot-wire anemometers was carried out *in situ* using a conventional Prandtl probe as a standard.

The probe carriage was attached to the mild steel strips inlet in the model by Teflon covered sintered Alnico magnets. This arrangement permitted chordwise traversing of the entire probe carriage with a remotely controlled electrical drive.

The hot-wire carriage may be seen in place on the model in figure 2 (plate 1).

Classification of the flow field

Extensive surveys made with the multiple hot-wire array revealed that the flow field produced by an adverse pressure gradient could be classified into three distinct regions. Proceeding in a streamwise direction, the first of these regions was characterized by the regularity and repetitive nature of the oscillating velocity wave form in the boundary layer. In this region random perturbations in the wave form were infrequent, and when observed, were quickly damped. When velocity fluctuations of this nature occurred, it was only in conjunction with similar perturbations in the free-stream velocity. This portion of the flow may be termed the 'pre-wake region'. The pre-wake region ends at a downstream point where perturbations on the oscillating velocity wave form begin to originate in the boundary layer, persist, and grow with downstream displacement. No associated velocity disturbances are observed in the free-stream flow. This region may be referred to as the 'region of wake formation'. Velocity perturbations in the region of wake formation increase in severity until they finally begin to induce significant alterations in the local free-stream velocity, marking the beginning of the 'region of wake flow'. Figure 4 (plate 2) presents a sequence of wave-form photographs along a boundary-layer mean streamline, illustrating the characteristics of these three distinct flow regions. In these photographs, as well as subsequent wave-form sequences, the upper oscilloscope sweep depicts the oscillating velocity component in the local free stream.†

Flow reversal

The multiple hot-wire array revealed that certain periodically occurring phenomena were consistently observed in the boundary-layer velocity wave forms. The most significant of these effects was characterized by an initial flattening of the wave form peak at the position of minimum local velocity. This first occurs in the lower half of the boundary layer. As the hot-wire is moved closer to the wall, the distortion intensifies, resulting in an apparent dip in the wave form, indicating a transient relative increase in velocity. This phenomenon was interpreted as an indication of the existence of reverse flow during the affected portion of the cycle based on the fact that the hot-wire anemometer is an absolute value sensor; hence for an instantaneous reverse flow condition, the anemometer will present the negative, or reversed, velocity portion of the wave form as positive, resulting in a 'folded' wave form. This deduction was verified by plotting the instantaneous profiles for that portion of the cycle, and observing that the wave-form dip must be considered as negative, or reversed, to preserve the continuity of the velocity derivatives. Figure 5 (plate 3) is a typical wave-form sequence, observed during a vertical traverse of the boundary layer, in which the reversal phenomena is evident. Since the hot-wire anemometer is an absolute

† In all oscillograms presented, the voltage, or velocity axis is inverted such that velocity increases progress in the 'negative' co-ordinate direction due to a voltage inversion inherent in the anemometer circuit. The time co-ordinate is oriented in the usual manner, i.e. increasing from left to right.

value sensor, the occurrence of flow reversal introduces an additive error in the measured mean velocity. This was corrected during the data reduction procedure.

In addition to the reversal-dip, two other periodically occurring phenomena were observed in the boundary-layer velocity wave forms. These were evident, in varying degrees, for most operating conditions, and are characterized as follows: 'amplification' – the growth, or amplification of the oscillatory wave form

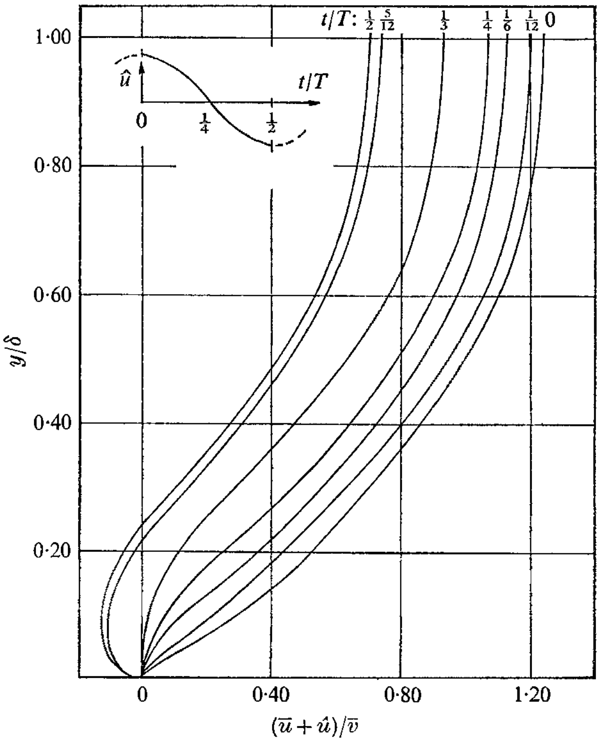

FIGURE 6. Typical sequence of instantaneous profiles during half-cycle of oscillation. Pre-wake region. Strong transient flow reversal. Run 24 (maximum to minimum free-stream velocity). $Re = 2{\cdot}745 \times 10^5$, $f = 5{\cdot}552 \times 10^{-5}$, $A = 0{\cdot}28$.

beyond its local free-stream magnitude as the boundary layer is vertically traversed; 'phase shift' – a lead, or tendency of the oscillatory changes in the boundary layer to occur earlier in time than respective changes in the free-stream flow. Both of these effects may be observed in figure 5. Amplification in this case is not pronounced, however a maximum phase lead of about 40 degrees is observed near the surface. Both of these effects have been analytically predicted by Lighthill and others.

Figure 6 illustrates typical behaviour of instantaneous velocity profiles at a point in the pre-wake region, during a half cycle of oscillation. These were obtained from the oscillograms shown in figure 5. It should be noted that in figure 6, the instantaneous boundary-layer velocity is normalized with respect to the local free-stream mean velocity, in order to make temporal profile variations readily apparent.

The profile sequence in figure 6 was obtained well upstream of the region of wake formation. It is apparent that a reverse flow condition exists during approximately one-half of the full cycle of oscillation, reaching its maximum reversal when the instantaneous free-stream velocity is at its minimum in the

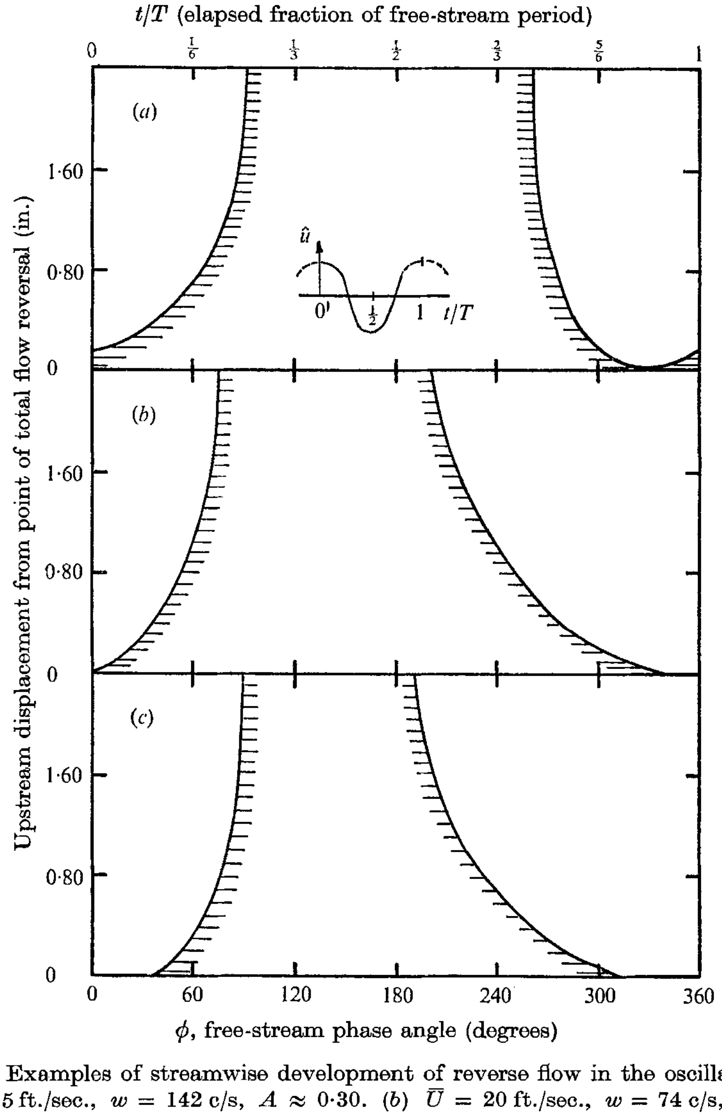

FIGURE 7. Examples of streamwise development of reverse flow in the oscillatory cycle. (*a*) $\bar{U} = 15$ ft./sec., $w = 142$ c/s, $A \approx 0{\cdot}30$. (*b*) $\bar{U} = 20$ ft./sec., $w = 74$ c/s, $A \approx 0{\cdot}25$. (*c*) $\bar{U} = 20$ ft./sec., $w = 226$ c/s, $A \approx 0{\cdot}20$.

cycle. Transient flow reversal of this nature was found to be present to some extent in all adverse pressure gradients. For amplitudes of oscillation of less than eight per cent of the local mean free-stream velocity, transient flow reversal first occurs at some point in the pre-wake region. For larger amplitudes, transient flow reversal was observed throughout the pre-wake region. This latter behaviour suggests that transient flow reversal may even occur in the zero pressure gradient region, as predicted by Stuart (1963) and Moore (1958) for sufficiently large amplitudes.

More significantly, it appears that for oscillatory flow the 'transition' region defined by Sandborn (1969) may actually extend well upstream of the adverse pressure gradient régime and contain two subregions ('pre-wake' and 'wake-formation'), based on the propagation of random disturbances.

Moving downstream, profile inflexion gradually became more pronounced throughout the cycle. The duration of transient flow reversal in the cycle, however, increased only slightly with downstream displacement. At a point generally between two and four boundary-layer thicknesses upstream of the region of wake formation, the reversed flow portion of the cycle began to expand rapidly as the 'least reversed' profile approached a zero velocity gradient condition at the wall. Finally, at a point within half a boundary-layer thickness of the beginning of the region of wake formation, the entire cycle of profiles exhibited reversed flow or zero velocity gradient at the wall. This behaviour was common to all operating conditions, and varied only in the magnitude of the initial cyclic fraction of reversed flow. Figure 7 illustrates the growth of the reverse flow portion of the cycle for several operating conditions.

Definition of separation

The primary goal of the present work was to evolve a rational definition of separation in oscillating flows and the findings presented here amply verify previous predictions that the steady flow definition of separation is inapplicable in oscillating flows. Reverse flow, and consequently zero wall shear profiles, were observed to occur periodically at all points on the surface in the adverse pressure gradient régime, upstream of the region of wake formation. These transient flow reversals caused no observed resultant perturbations either in the boundary layer or the free stream. Further, it may reasonably be assumed that transient flow reversal occurs even in the zero pressure gradient régime for many operating conditions. This phenomenon clearly precludes the use of the steady flow definition, since no intial occurrence of a zero wall shear profile or flow reversal can be rigorously related to wake formation.

The results of the present work reveal that only a single observable flow phenomenon is clearly related to the development of wake flow. This is the initial occurrence of zero velocity or reverse flow instantaneous profiles throughout the entire cycle of oscillation, or 'continuous flow reversal'. For all operating conditions observed, the region of wake formation invariably began in the immediate neighbourhood of the initial point of continuous flow reversal. Moreover, it was found that this point could be rigorously and uniquely located for all observed operating conditions.

The following definition is therefore proposed: Laminar boundary-layer separation in oscillating flows is defined as commencing with the initial occurrence of zero velocity or reverse flow at some point in the velocity profile throughout the entire cycle of oscillation.

This definition has the following important features: (1) The phenomenon described has been experimentally observed to be directly related to the production of wake flow. (2) Separation, thus defined, occurs at a unique point in the flow field. (3) The definition may be extended to steady flow with no ambiguity. (4) The definition is useful in analytic as well as experimental investigation.

A suitable criterion for the recognition of separation is the observation of a velocity gradient at the wall which is less than or equal to zero throughout the entire cycle of oscillation.

$$du/dy \leqslant 0 \quad \text{or} \quad y = 0.$$

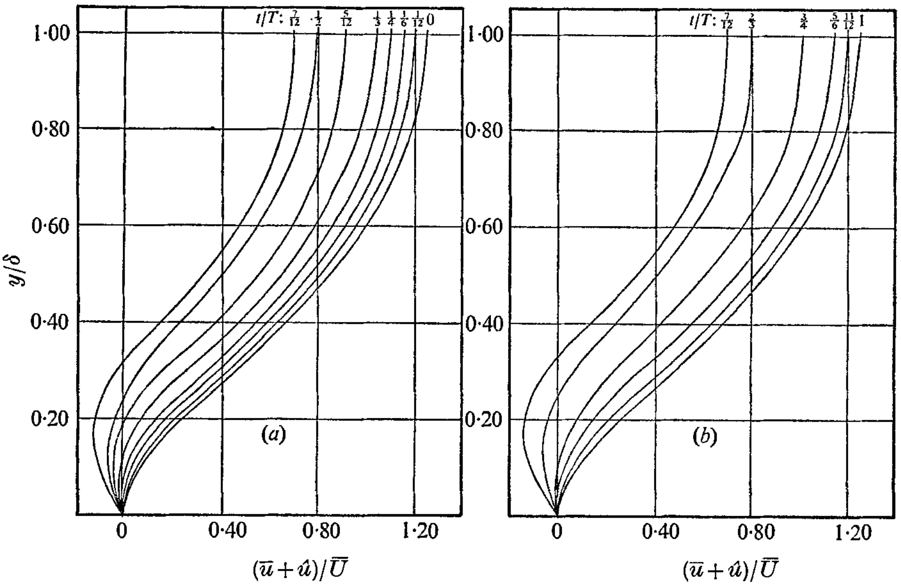

FIGURE 8. Typical cycle of instantaneous velocity profiles at separation. Strong flow reversal. Run 18. (*a*) Maximum to minimum free-stream velocity. (*b*) Minimum to maximum free-stream velocity. $Re = 2{\cdot}235 \times 10^5$, $P = 0{\cdot}2283$, $f = 9{\cdot}871 \times 10^{-5}$, $A = 0{\cdot}32$.

Figure 8 illustrates typical cyclic variation of the instantaneous velocity profile at separation. Figure 9 (plate 4) presents the associated wave-form photographs. Rigorous definitions of the dimensionless pressure gradient, frequency and amplitude parameters, N_P, N_f and N_A, are presented below.

These results are representative of 27 sets of separation point data obtained with the initial pressure distribution. Once the separation criterion was established from this information, the pressure distribution on the model was varied and an additional 36 separation points located. Steady flow separation points were located in the usual manner for each of the combinations of mean velocity and pressure distribution employed in the investigations, allowing the separation point displacement to be obtained.

Correlation of separation point alteration

The second goal of the present work was the correlation of data in a meaningful way which would permit the engineering prediction of stall in oscillating flows. It was discovered that the key to this problem lay in adequately characterizing the boundary-layer pressure-gradient history. Through a combination of empirical and intuitive means it was found that the parameter

$$P = \left[\int_{x_0}^{x_s} x\frac{d\bar{p}}{dx}dx \bigg/ \int_0^{x_s} x\left|\frac{d\bar{p}}{dx}\right|dx\right]\left(\frac{dp}{dx}\right)_{x_s}\frac{x_0}{q},$$

where $\bar{p}$ = mean static pressure, q = free-stream dynamic pressure, x = length co-ordinate, x_0 = point at which the pressure gradient becomes positive for the last time prior to separation, x_s = separation point, allowed all of the data from 63 different flow conditions to be correlated within 3 % in the form:

$$\Delta_s = 1{\cdot}13\times 10^7 Re^{-1{\cdot}77} P^{-0{\cdot}28} f^{-0{\cdot}26} A^{0{\cdot}035},$$

where Δ_s = shift in separation point from its steady flow location, $(x_{ss}-x_s)/x_{ss}$, Re = Reynolds number, Ux_s/ν, f = oscillation frequency parameter, $\omega\nu/U^2$, A = oscillation amplitude parameter, $\Delta U/U$, ΔU = oscillation amplitude, U = mean free-stream velocity, x_{ss} = steady flow separation point, ω = oscillation frequency, ν = kinematic viscosity.

Although one might, *a priori*, expect the dominant parameter to be the Reynolds number, as indeed it turns out, it is somewhat surprising to find that the oscillation amplitude as characterized by the amplitude parameter, A, has a negligible influence on separation-point displacement.

Conclusions

From the results, the following conclusions may be drawn: (1) The first indications of wake formation occur at that streamwise point in the flow where the normal velocity gradient at the wall is observed, for the first time, to be less than or equal to zero throughout the entire cycle of oscillation. This point may then be logically defined as the 'separation point', marking the beginning of boundary-layer separation. (2) The presence of an oscillating velocity component, superimposed on the mean flow, causes separation to occur upstream of the steady flow separation point. (3) Increasing frequency of oscillation tends to cause the separation point to move downstream toward the steady flow location. (4) For the range of flow parameters observed, the effects of amplitude variation on separation point location appear negligible. (5) Transient boundary-layer flow reversal occurs throughout the adverse pressure gradient régime for amplitude parameters of the order of 0·10 or greater, without inducing wake formation, confirming the predictions of Stuart (1963), Moore (1958) and Sandborn (1969).

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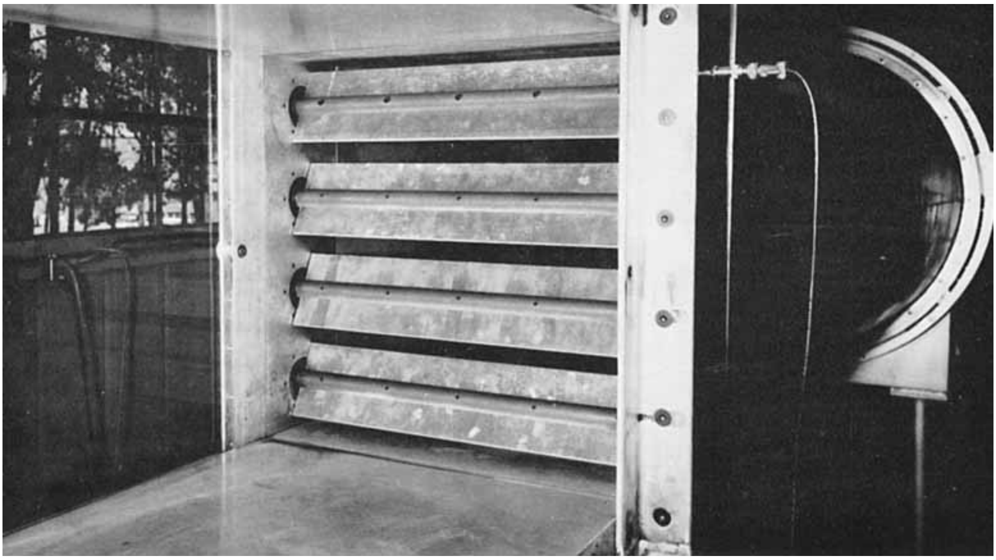

FIGURE 1. The rotating shutter value.

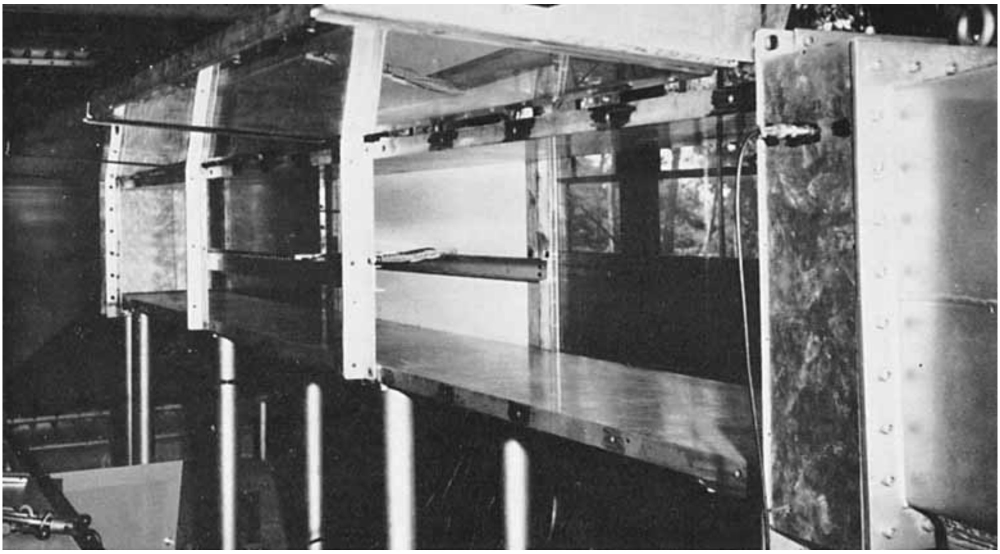

FIGURE 2. The test section.

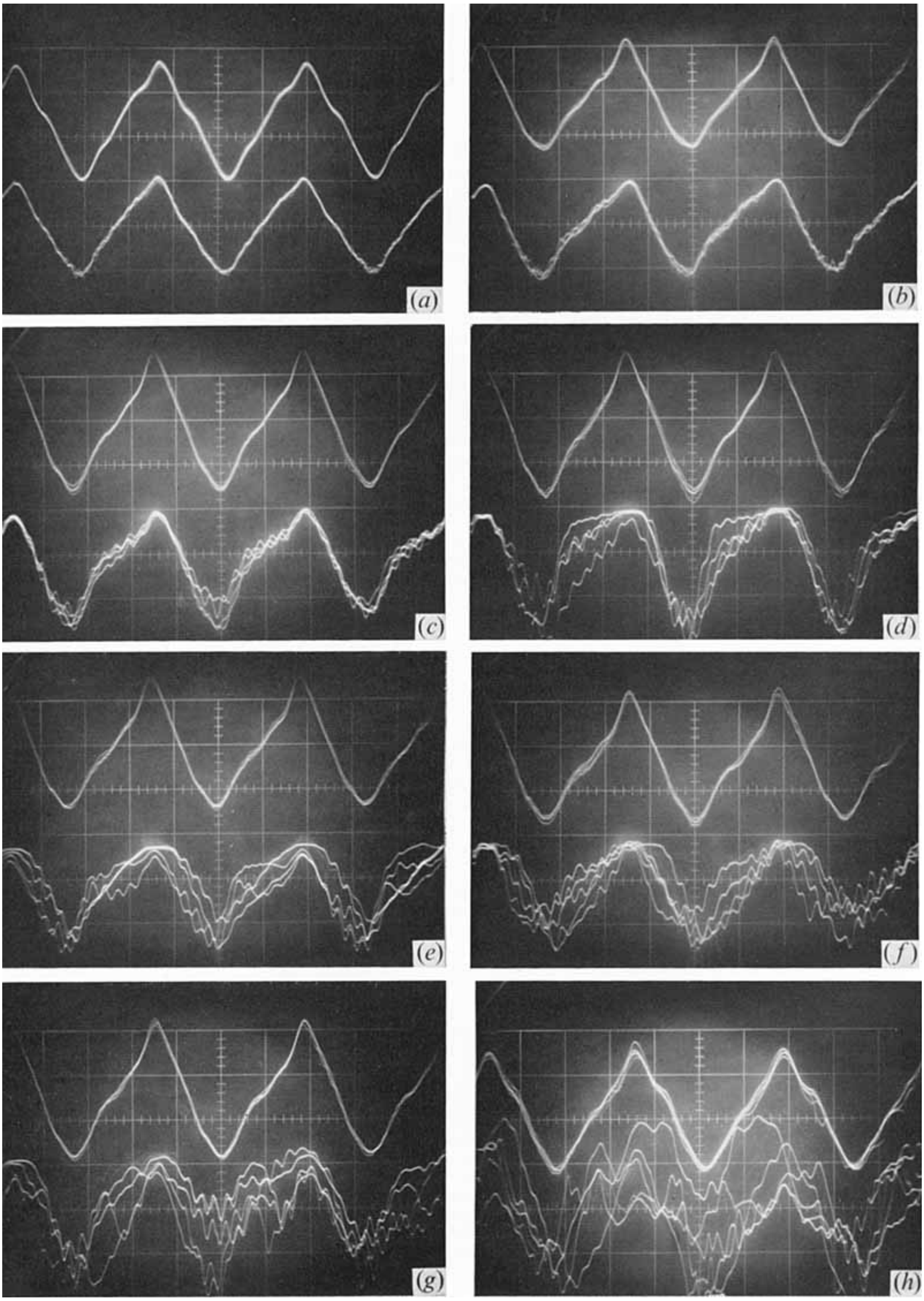

FIGURE 4. Typical velocity wave forms in the three adverse pressure-gradient flow regions. $\overline{u}/\overline{U} = 0{\cdot}40$, $f = 8{\cdot}837 \times 10^{-5}$, $A \approx 0{\cdot}36$. (*a*) Pre-wake, $Re = 2{\cdot}745 \times 10^5$, $x/c = 0{\cdot}616$. (*b*) Begin wake formation, $Re = 2{\cdot}809 \times 10^5$, $x/c = 0{\cdot}626$. (*c*) Wake formation, $Re = 2{\cdot}863 \times 10^5$, $x/c = 0{\cdot}640$. (*d*) Wake formation, $Re = 2{\cdot}903 \times 10^5$, $x/c = 0{\cdot}648$. (*e*) Wake formation, $Re = 2{\cdot}927 \times 10^5$, $x/c = 0{\cdot}655$. (*f*) Wake formation, $Re = 2{\cdot}959 \times 10^5$, $x/c = 0{\cdot}662$. (*g*) Wake formation, $Re = 2{\cdot}991 \times 10^5$, $x/c = 0{\cdot}669$. (*h*) Begin wake flow, $Re = 3{\cdot}023 \times 10^5$, $x/c = 0{\cdot}676$.

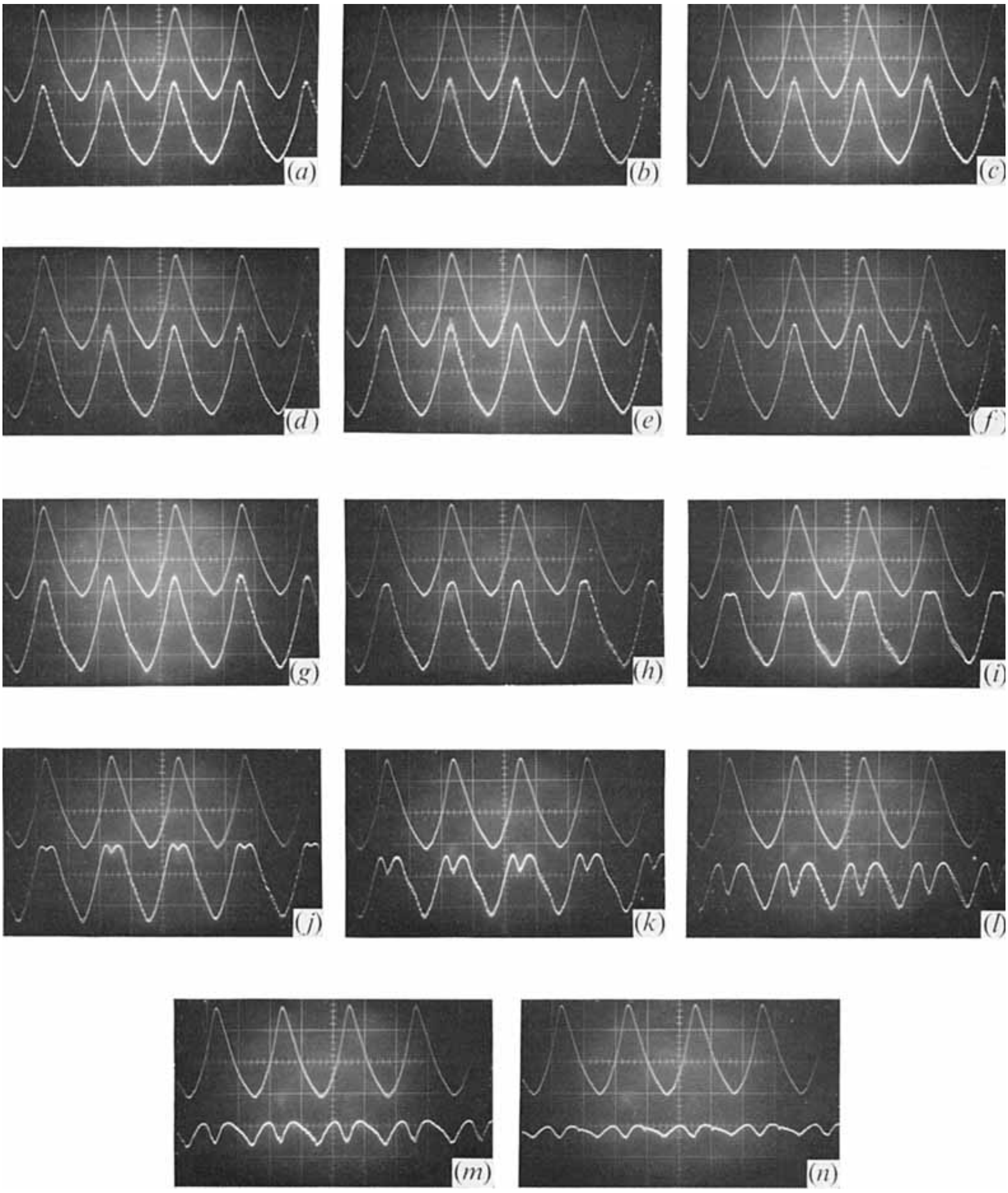

FIGURE 5. Oscillating velocity component wave forms. Run 2, $Re = 2{\cdot}745 \times 10^5$, $f = 5{\cdot}552 \times 10^{-5}$, $A = 0{\cdot}28$. Pre-wake region. Major grid divisions: 5 msec × 0·2 volts. Values of y/δ: (*a*) 1·00; (*b*) 0·74; (*c*) 0·67; (*d*) 0·55; (*e*) 0·47; (*f*) 0·41; (*g*) 0·33; (*h*) 0·28; (*i*) 0·23; (*j*) 0·19; (*k*) 0·15; (*l*) 0·11; (*m*) 0·06; (*n*) 0·04.

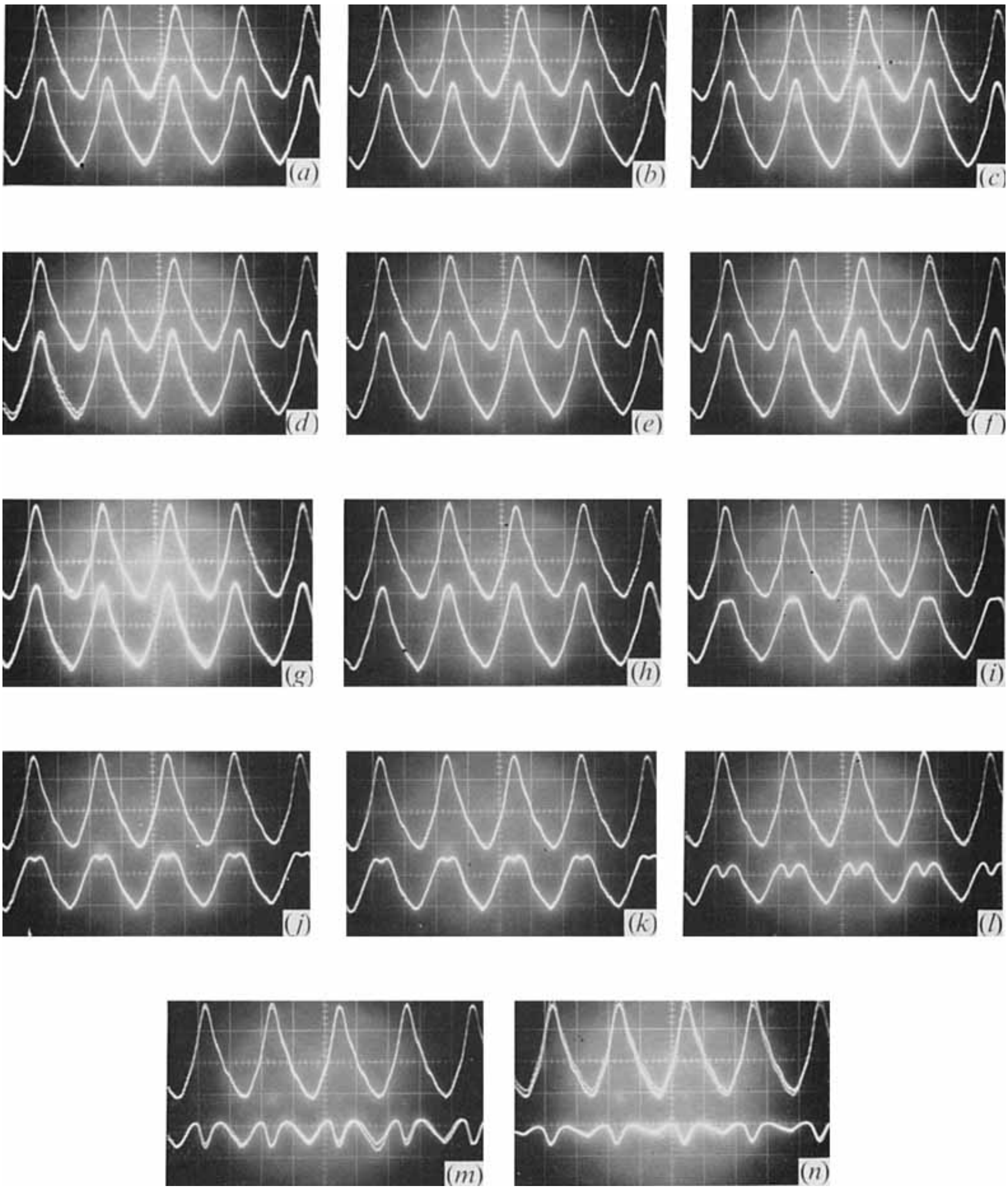

FIGURE 9. Oscillating velocity component wave forms at separation. Strong reverse flow. Run 18, $Re = 2{\cdot}235 \times 10^5$, $f = 9{\cdot}871 \times 10^{-5}$, $P = 0{\cdot}2283$, $A = 0{\cdot}32$. Major grid divisions: 5 msec $\times$ 0·2 volts. Values of y/δ: (*a*) 1·00; (*b*) 0·81; (*c*) 0·75; (*d*) 0·67; (*e*) 0·56; (*f*) 0·50; (*g*) 0·42; (*h*) 0·37; (*i*) 0·33; (*j*) 0·30; (*k*) 0·28; (*l*) 0·25; (*m*) 0·22; (*n*) 0·14.